



# Fluid Techniques

**Jim Van Verth**

Senior Engine Programmer, Insomniac Games

GAME DEVELOPERS CONFERENCE

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EXPO DATES: MARCH 7-9

**2012**

# Introductory Bits

- General summary with some details
- Not a fluids expert
- Theory and examples

# What is a Fluid?

- Deformable
- Flowing
- Examples
  - Smoke
  - Fire
  - Water

# What is a Fluid?



# What is a Fluid?



# What is a Fluid?



# Fluid Concepts

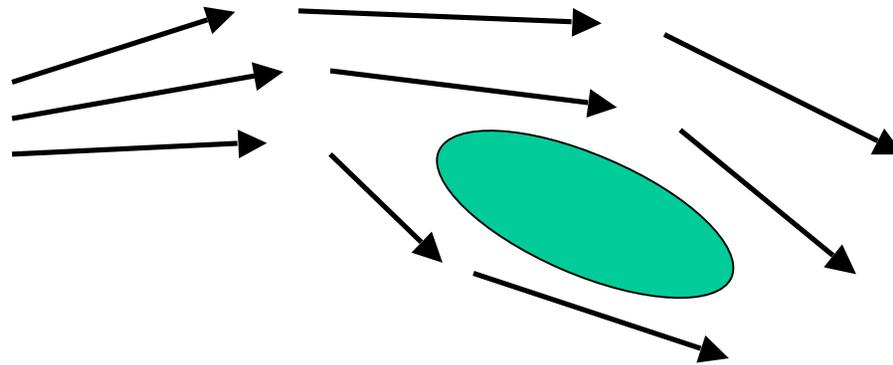
- Fluids have variable density
  - (Density field)



# Fluid Concepts

- Fluids "flow"

- (Vector field)

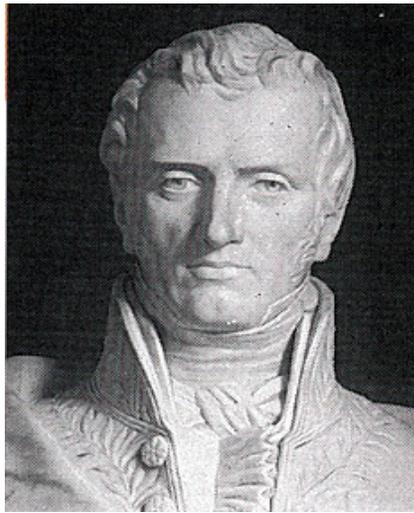


# Fluid Concepts

- Need way to represent
  - Density ( $\rho$ )
  - Velocity ( $\mathbf{u}$ )
  - Sometimes temperature

# Fluid Concepts

●Our heroes:



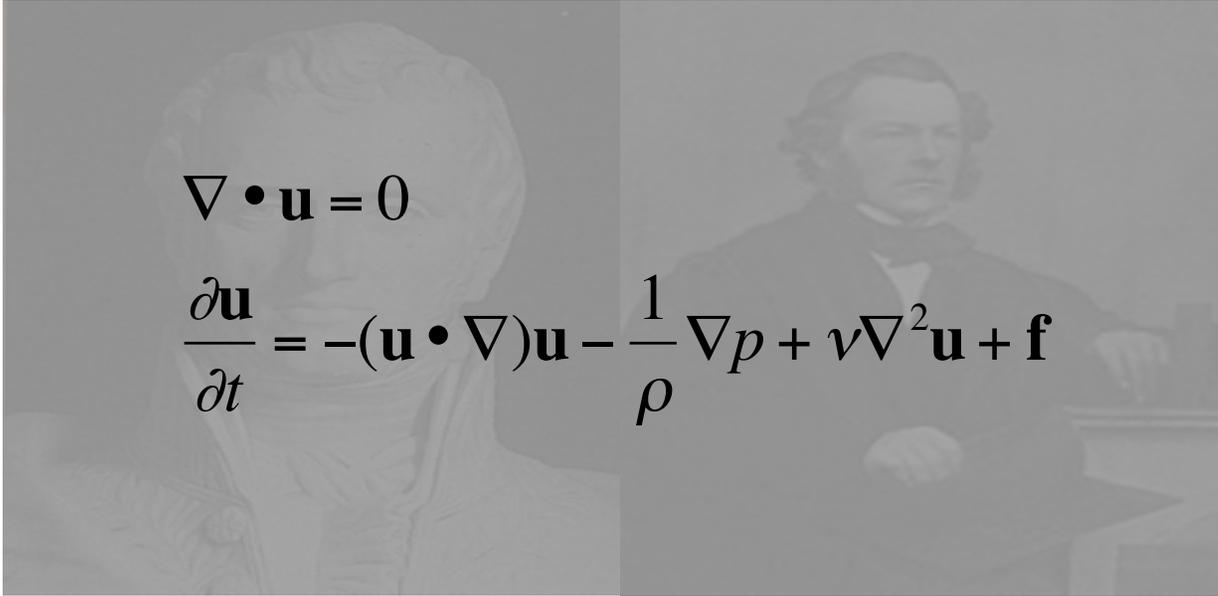
Navier



Stokes

# Fluid Concepts

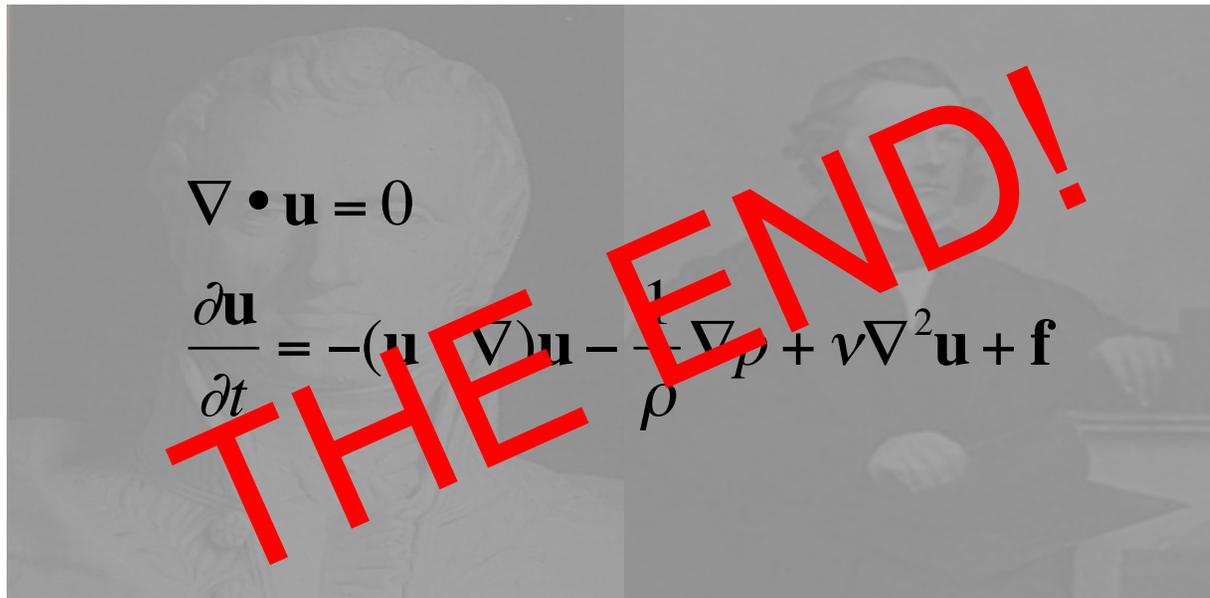
- Their creation:


$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

# Fluid Concepts

- Their creation:



# Fluid Concepts

- Their creation:

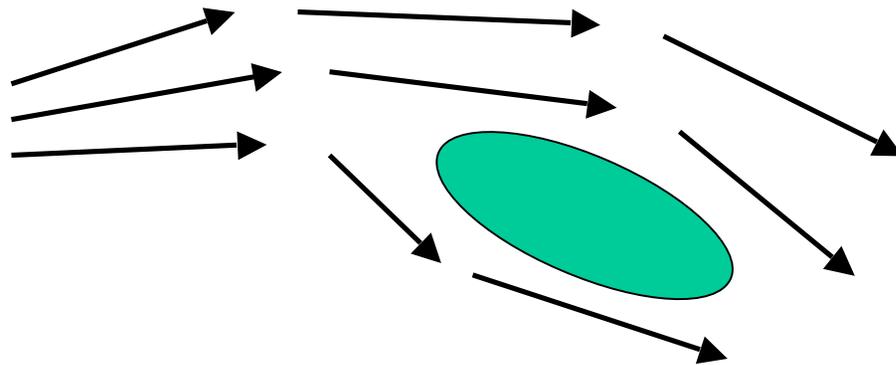
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$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

**SERIOUSLY --  
WHAT DOES THIS MEAN?**

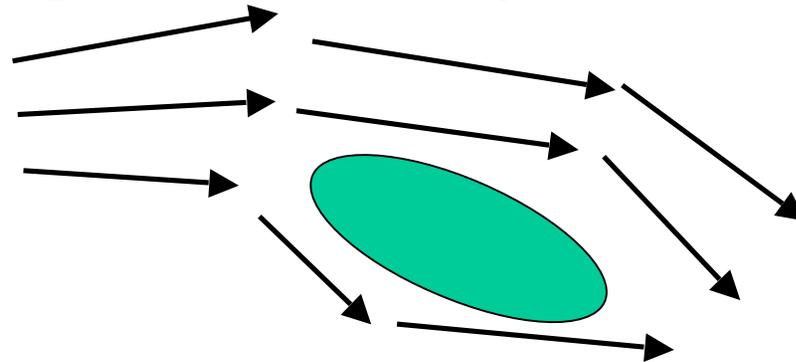
# Fluid Concepts

- Want change in velocity field



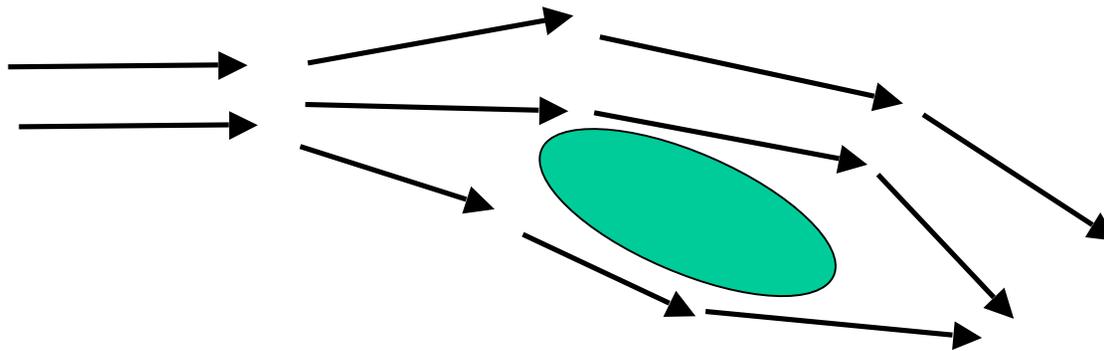
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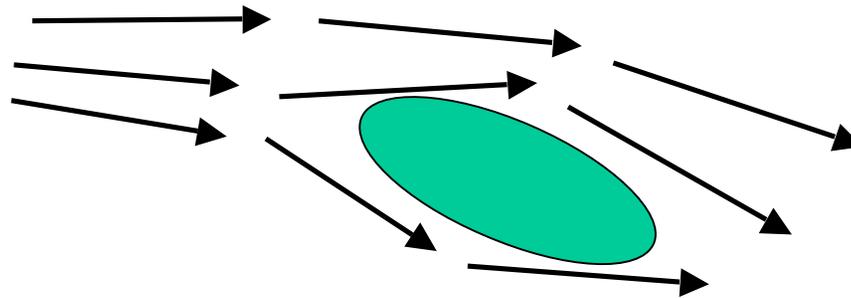
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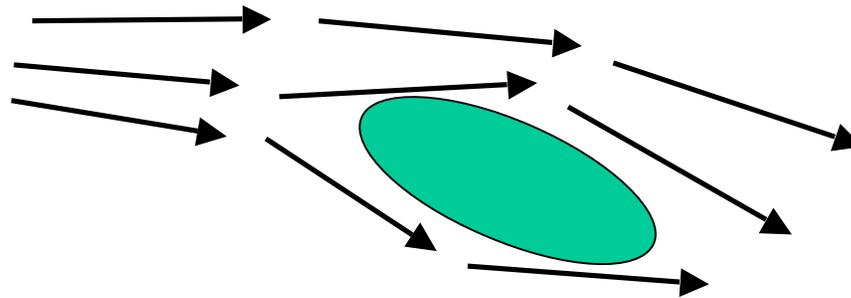
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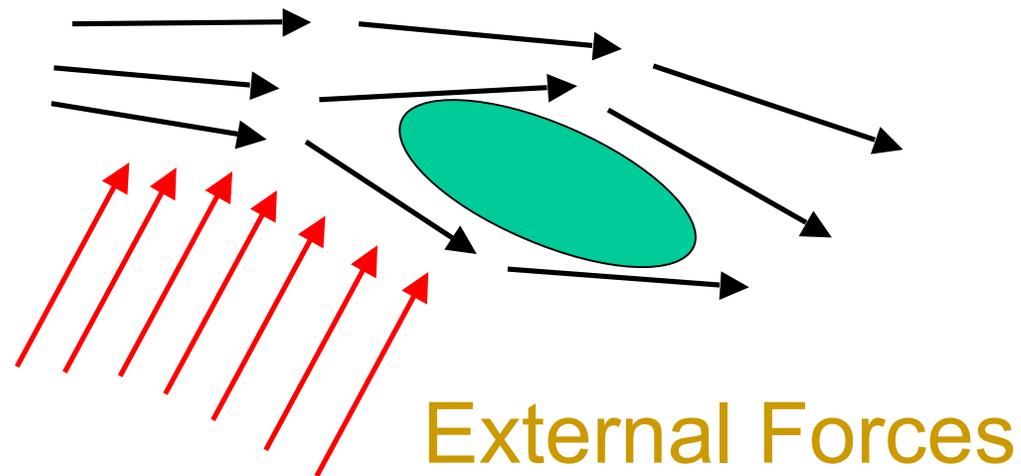
# Fluid Concepts

- What affects it?



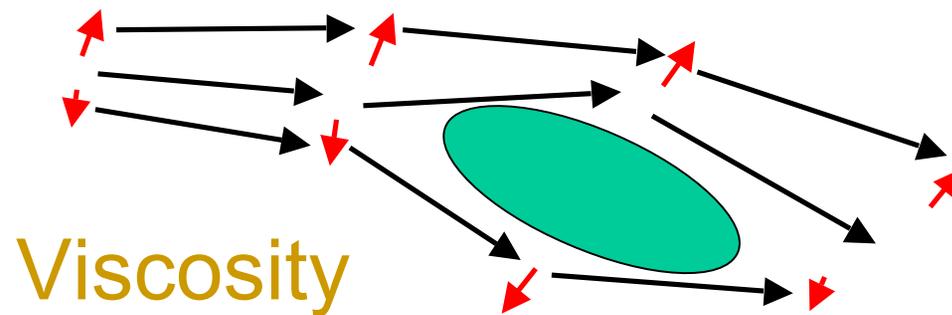
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- What affects it?



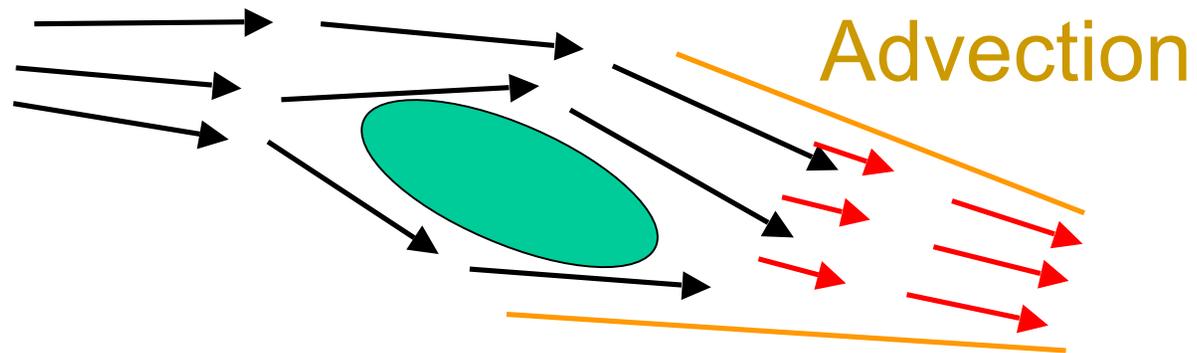
# Fluid Concepts

- What affects it?



# Fluid Concepts

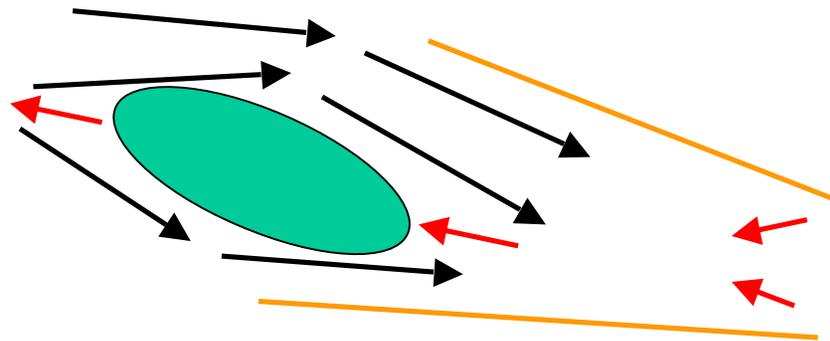
- What affects it?



# Fluid Concepts

- What affects it?

Pressure



# Fluid Concepts

## ● Back to Navier-Stokes

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

# Fluid Concepts

## ● Back to Navier-Stokes

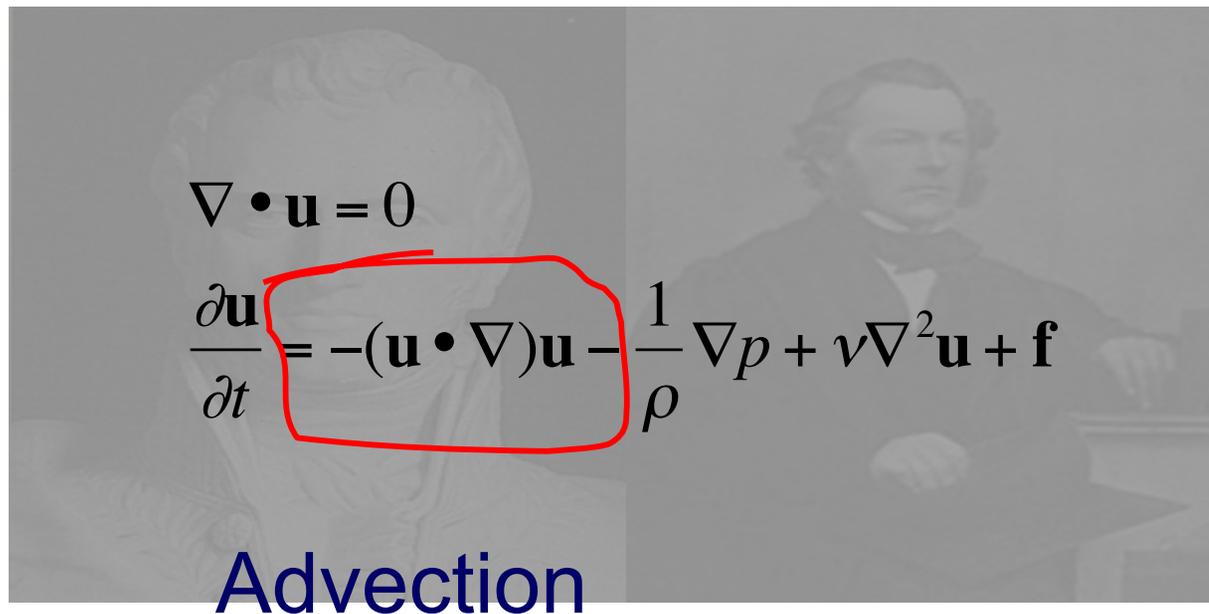
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$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

Change in Velocity

# Fluid Concepts

## ● Back to Navier-Stokes


$$\nabla \cdot \mathbf{u} = 0$$
$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

Advection

# Fluid Concepts

## ● Back to Navier-Stokes

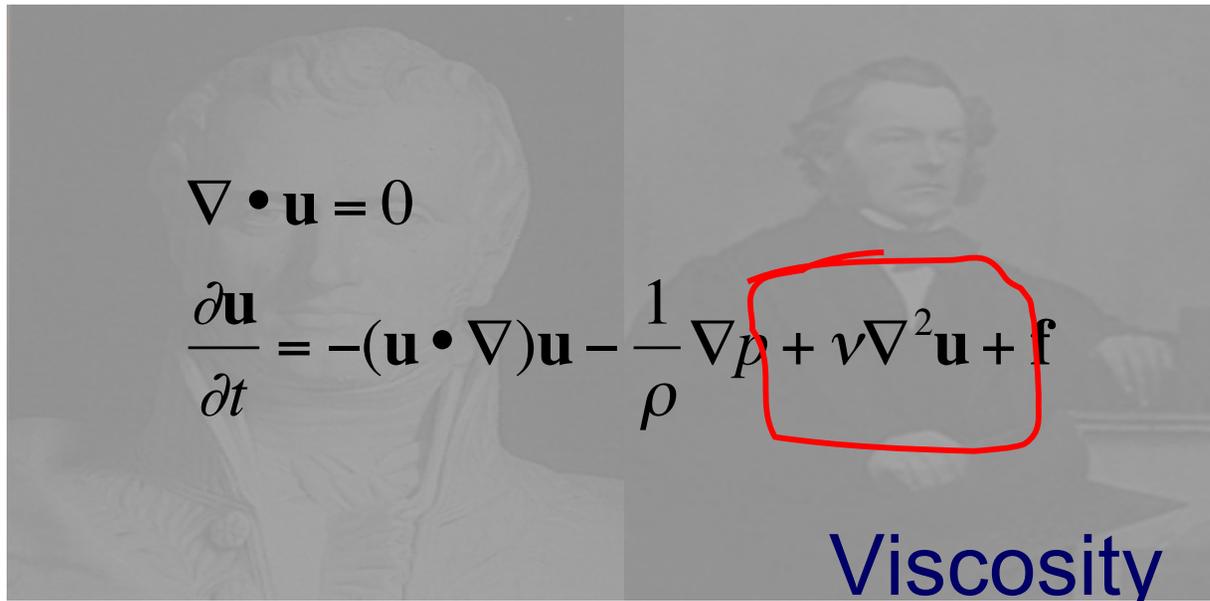
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Pressure

# Fluid Concepts

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$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

Viscosity

# Fluid Concepts

## ● Back to Navier-Stokes

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

External Forces

# Fluid Concepts

## ● Back to Navier-Stokes

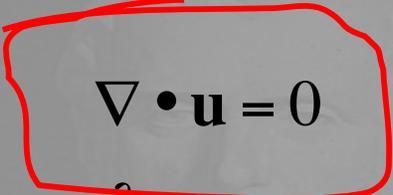
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$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

HOLD ON THERE BUCKO...

# Fluid Concepts

- Back to Navier-Stokes

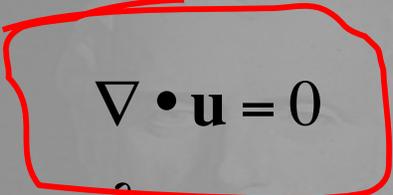

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$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

WHAT'S THIS ONE?

# Fluid Concepts

## ● Back to Navier-Stokes

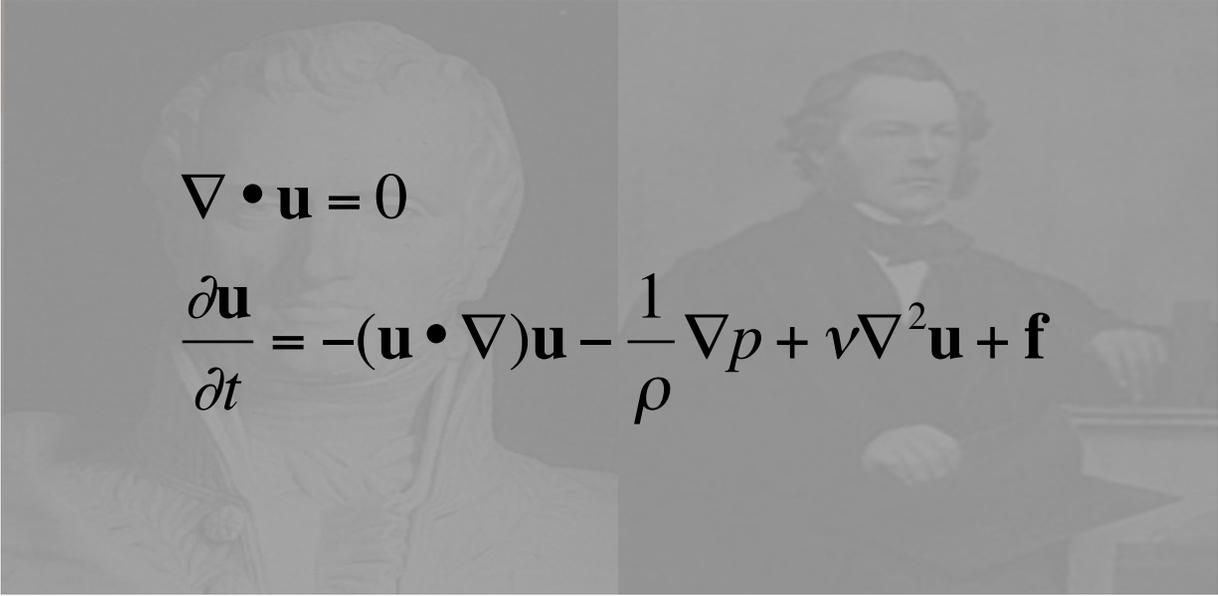

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

Mass Conservation

# Fluid Concepts

- In principle then, Navier-Stokes is...


$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

# Fluid Concepts

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**THE END!**

# Fluid Concepts

- But not really, of course

# Fluid Concepts

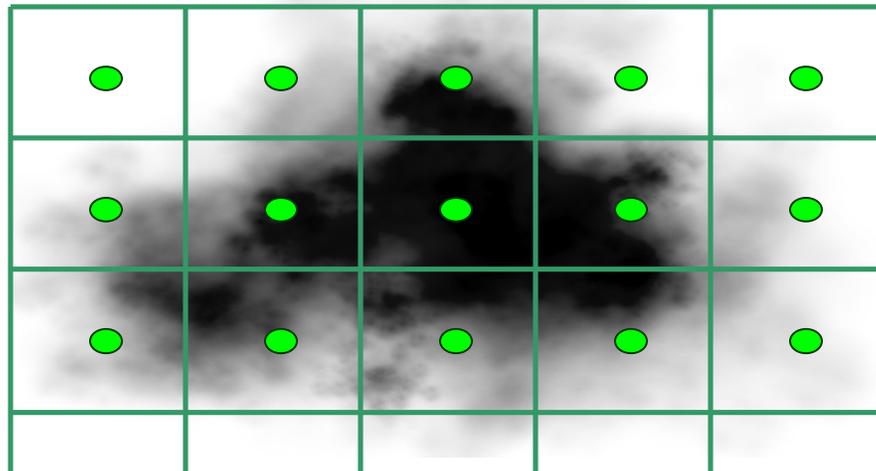
- But not really, of course
- Little tiny detail of implementation

# Computational Fluid Types

- Grid-based/Eulerian (Stable Fluids)
- Particle-based/Lagrangian (Smoothed Particle Hydrodynamics)
- Surface-based (wave composition)

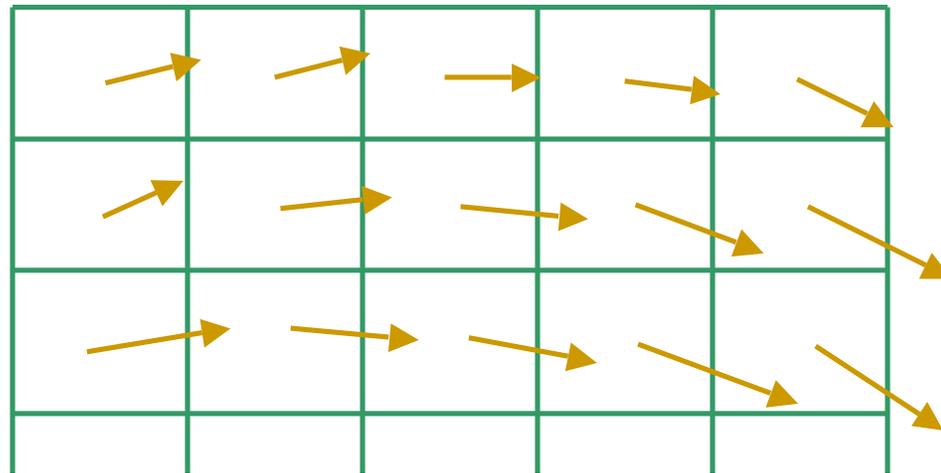
# Grid-Based

- Store density, temp in grid centers



# Grid-Based

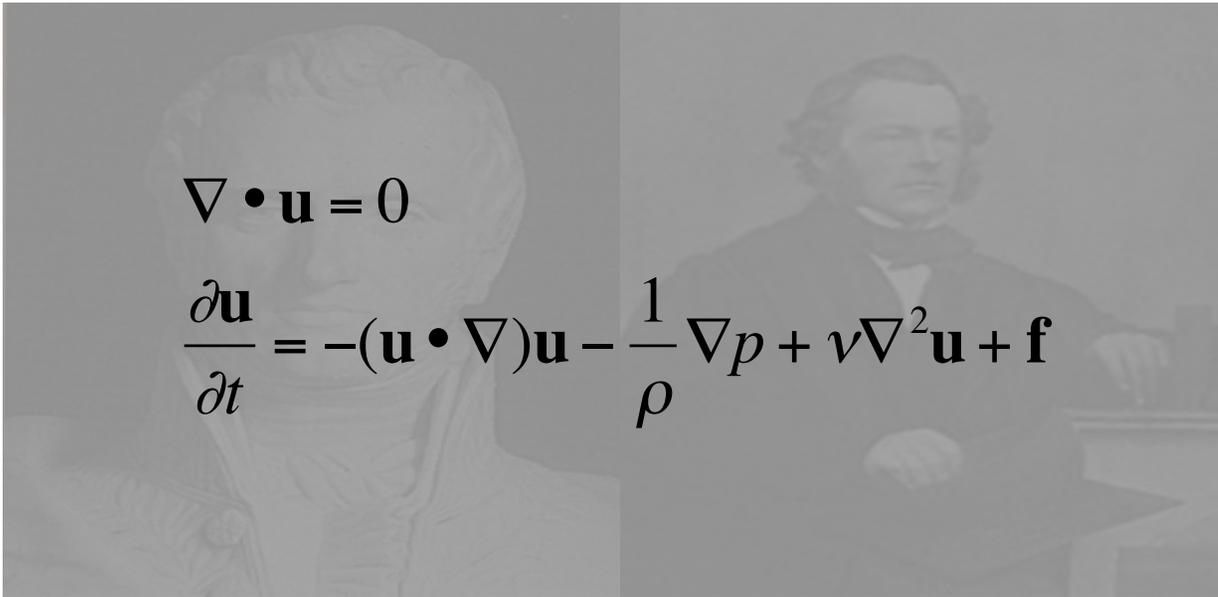
- Velocity (flow) from centers as well



- Could also do edges

# Grid-Based

- How do we use this?


$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

# Grid-Based

- Jos Stam devised stable approximation: "Stable Fluids", SIGGRAPH '99

# Grid-Based

- How do we use this?

$\nabla \cdot \mathbf{u} = 0$  Must maintain

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

# Grid-Based

- How do we use this?

$\nabla \cdot \mathbf{u} = 0$  Idea: compute

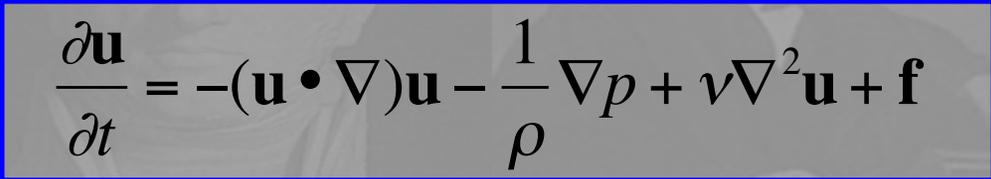
$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

# Grid-Based

- How do we use this?


$$\nabla \cdot \mathbf{u} = 0$$

Idea: compute


$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

# Grid-Based

- How do we use this?

$\nabla \cdot \mathbf{u} = 0$  Idea: compute

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

then project to 0 div field

# Grid-Based

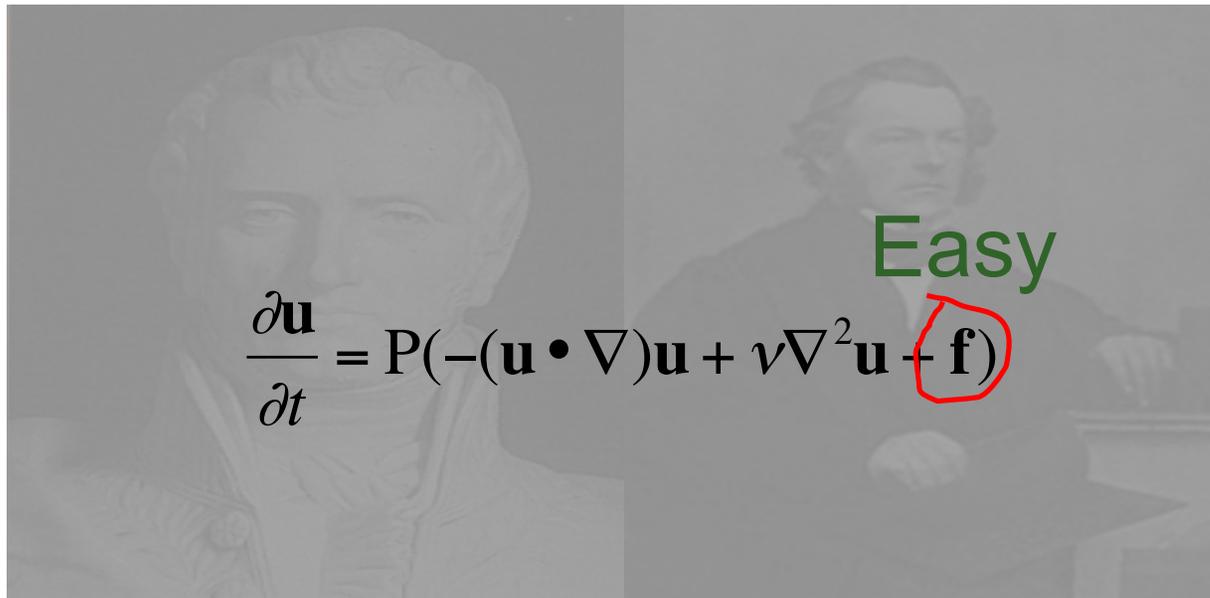
- How do we use this?

End up with

$$\frac{\partial \mathbf{u}}{\partial t} = P(-(\mathbf{u} \cdot \nabla) \mathbf{u} + \nu \nabla^2 \mathbf{u} + \mathbf{f})$$

# Grid-Based

- How do we use this?



# Grid-Based

- How do we use this?

Sparse linear system

$$\frac{\partial \mathbf{u}}{\partial t} = P(-(\mathbf{u} \cdot \nabla)\mathbf{u} + \nu \nabla^2 \mathbf{u} + \mathbf{f})$$

# Grid-Based

- How do we use this?

Sparse linear system

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{P}(-(\mathbf{u} \cdot \nabla)\mathbf{u} + \nu \nabla^2 \mathbf{u} + \mathbf{f})$$

# Grid-Based

- How do we use this?

Non-linear... ugh

$$\frac{\partial \mathbf{u}}{\partial t} = \rho(-(\mathbf{u} \cdot \nabla)\mathbf{u} + \nu \nabla^2 \mathbf{u} + \mathbf{f})$$

# Grid-Based

- How do we use this?

Non-linear... ugh

$$\frac{\partial \mathbf{u}}{\partial t} = P(-(\mathbf{u} \cdot \nabla) \mathbf{u} + \nu \nabla^2 \mathbf{u} + \mathbf{f})$$

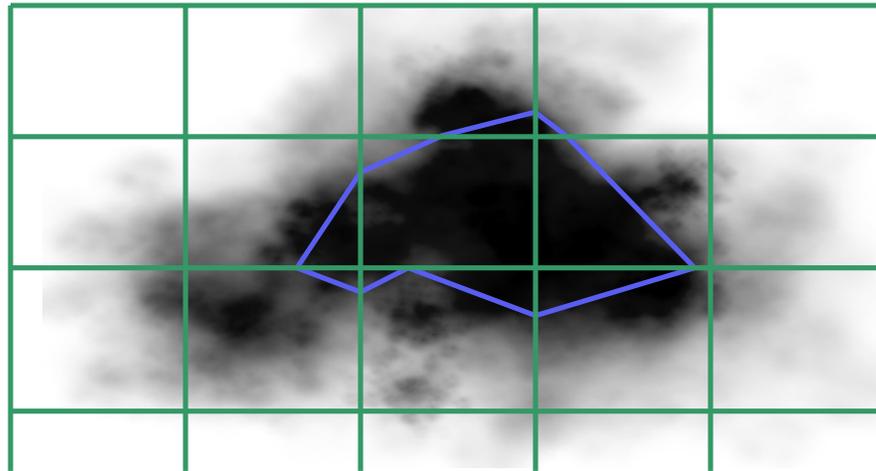
Can approximate

# Grid-Based

- Overview
  - Update velocities based on
    - Forces, then
    - Advection, then
    - Viscosity
  - Project velocities to zero divergence
  - Update densities based on
    - Input sources
    - Velocity
    - Diffusion (similar to viscosity, sometimes not used)
  - Draw it

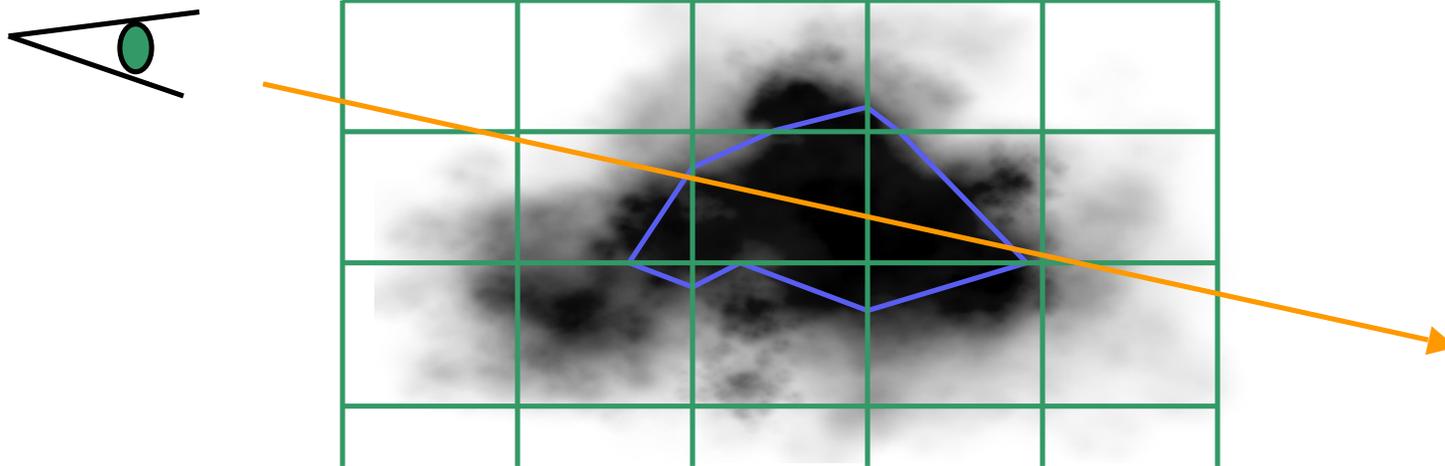
# Rendering Grid-Based

- Build level surface



# Rendering Grid-Based

- Determining color, transparency



# Issues

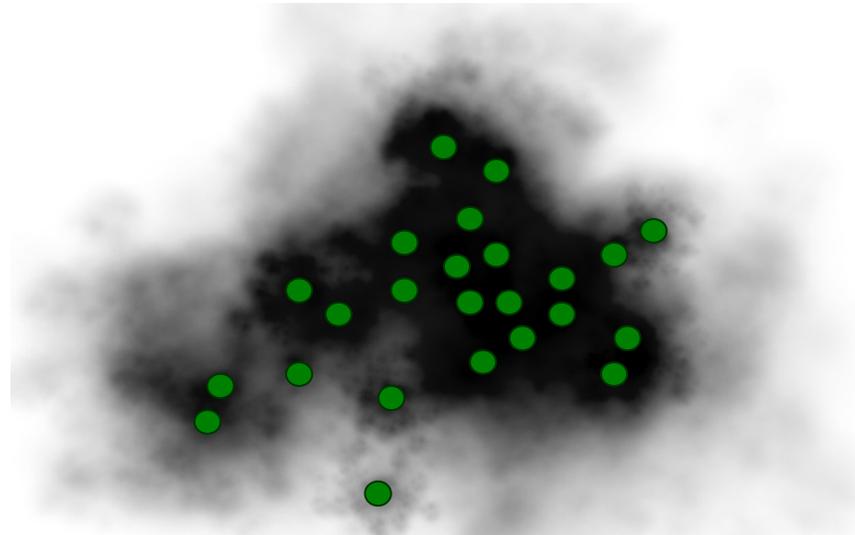
- Limited space
- Water “splashes” get lost
- Can be computationally expensive
- Dampens down
- But stable

# Implementation

- [Little Big Planet](#)
  - “Death smoke”
  - Bubble pop
  - Other smoke effects
- [Hellgate: London](#)
- [GDC09 NVIDIA demo](#)

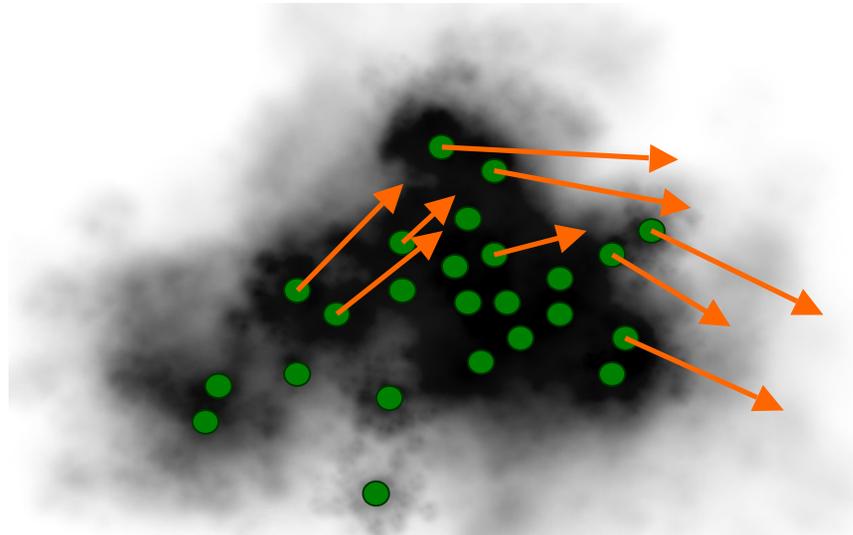
# Smoothed Particle Hydrodynamics

- Approximate fluid with small(er) set of particles



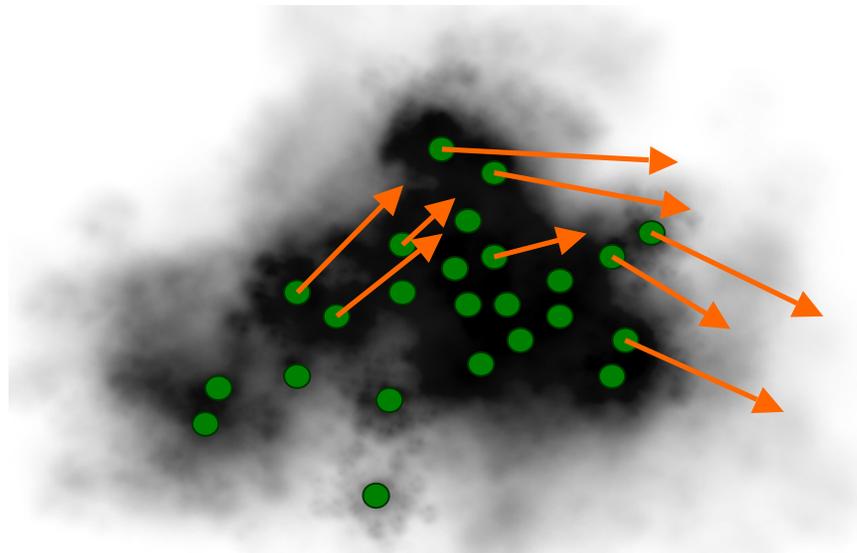
# SPH

- Velocities at particles provide flow



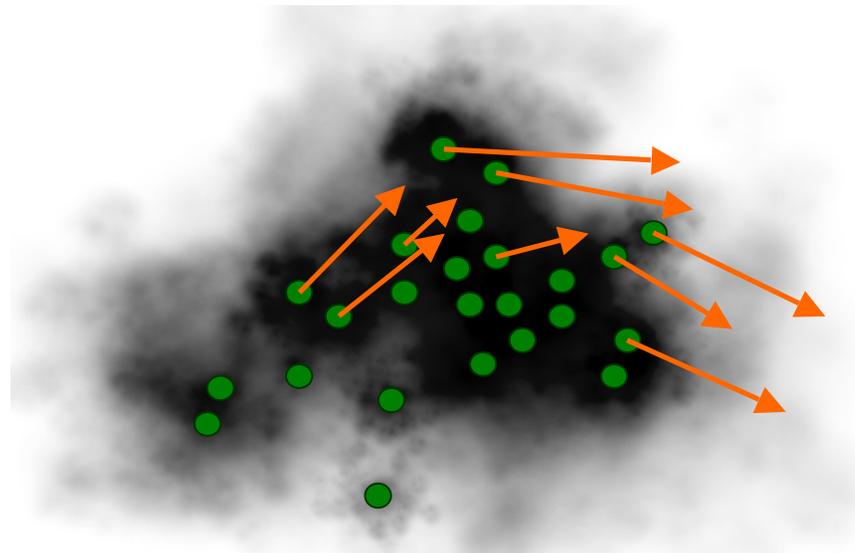
# SPH

- Idea: treat as particle system



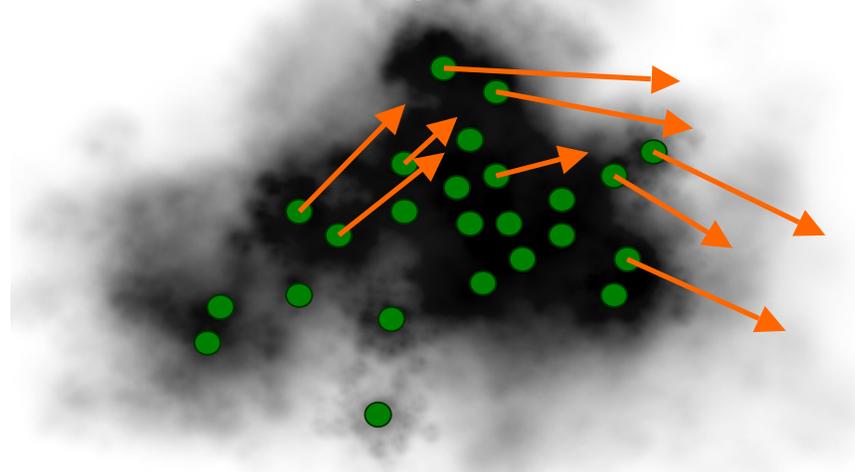
# SPH

- Idea: treat as particle system
  - Determine forces



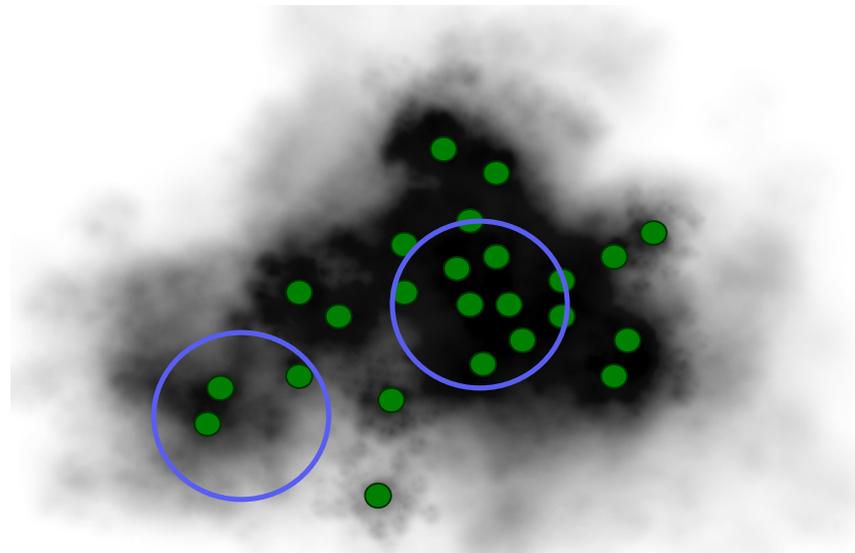
# SPH

- Idea: treat as particle system
  - Determine forces
  - Update velocities, positions



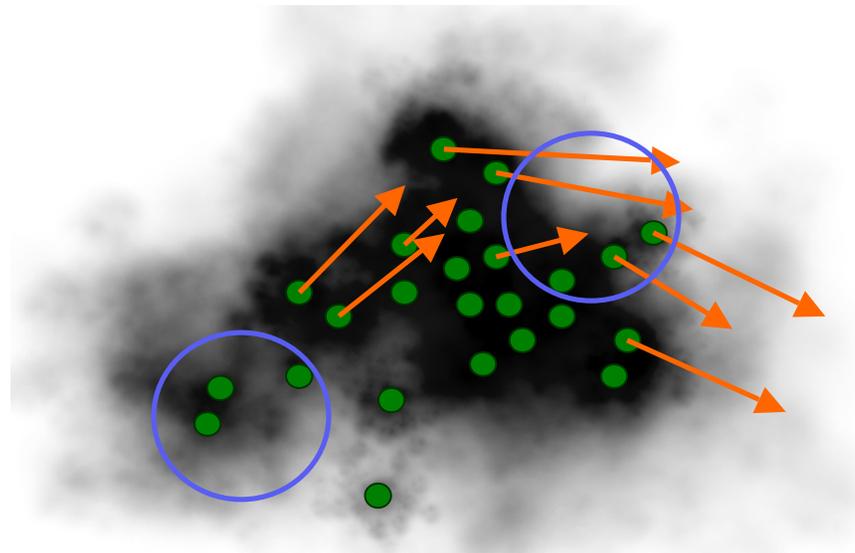
# SPH

- Weighted average gives density (smoothing kernel)



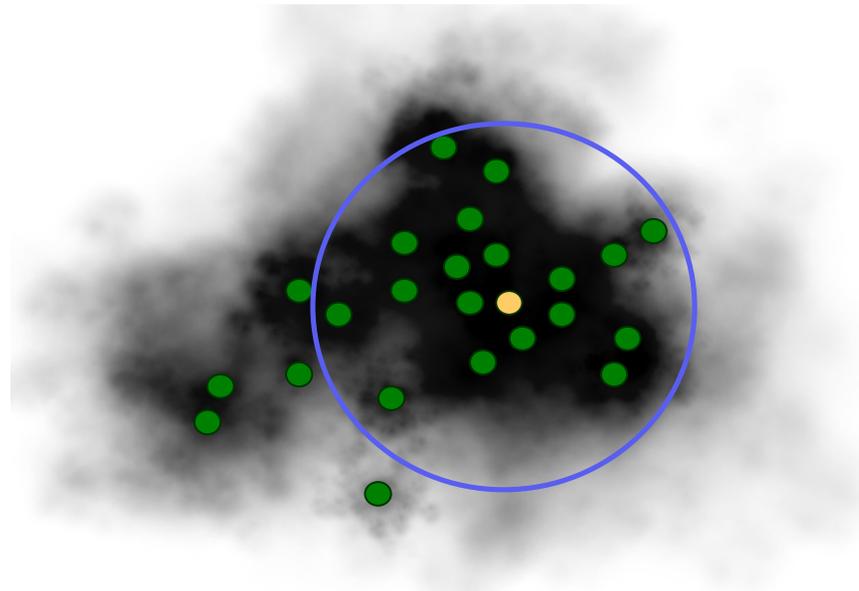
# SPH

- Can also use kernel to get general velocity



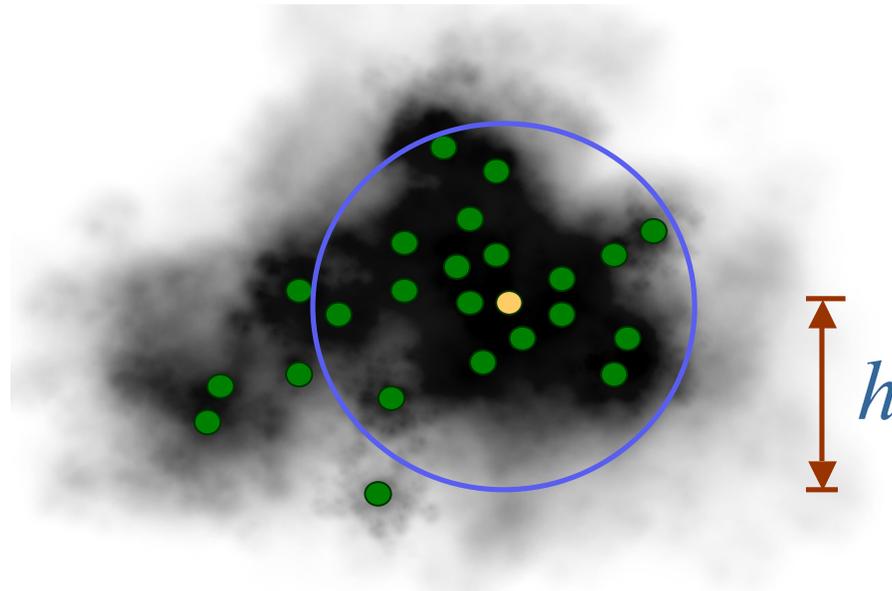
# SPH

- Usually center at particle



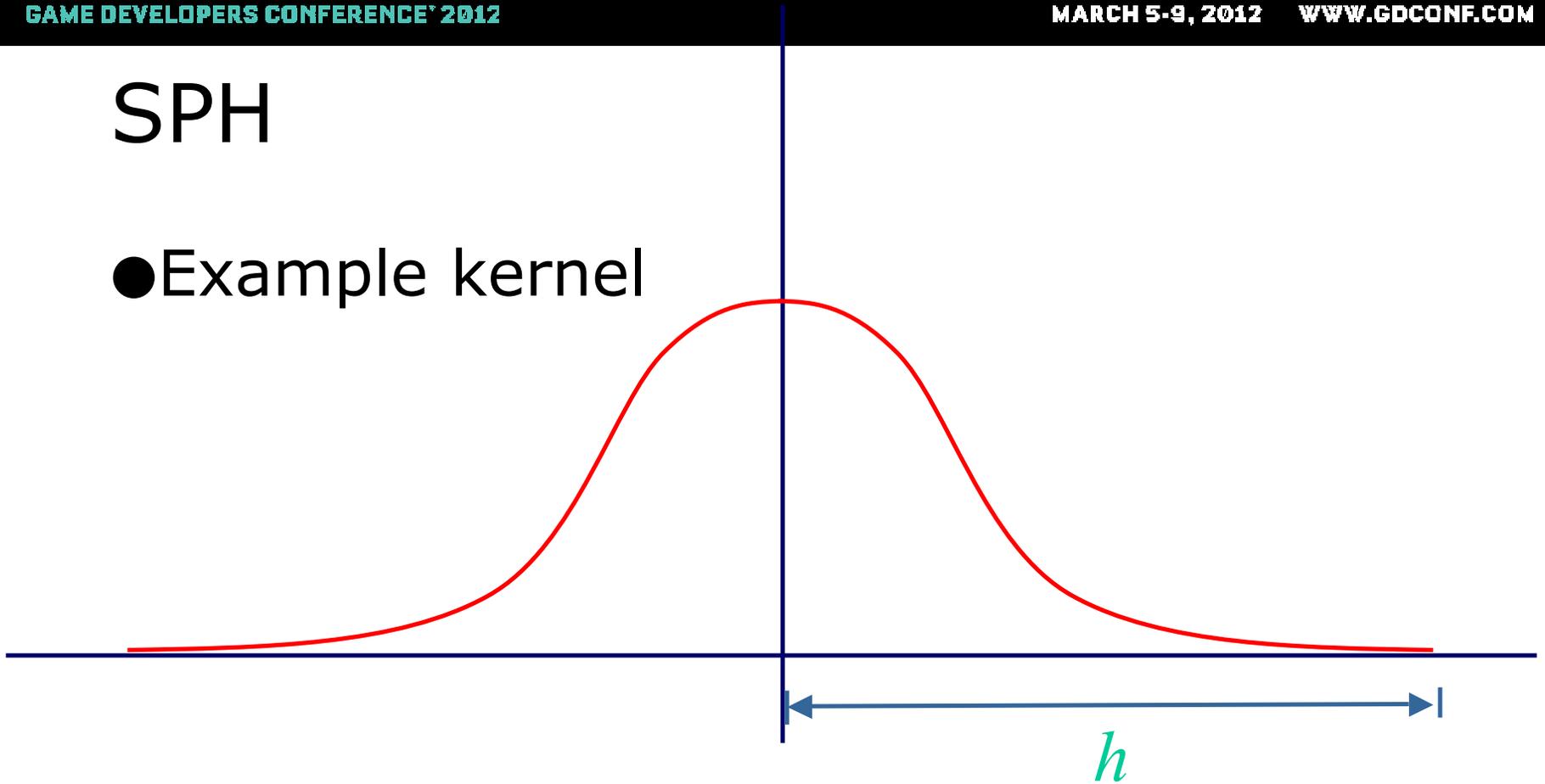
# SPH

- Specify width by  $h$



# SPH

## ● Example kernel



# SPH

## ● Back to Navier-Stokes

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# SPH

## ● Back to Navier-Stokes

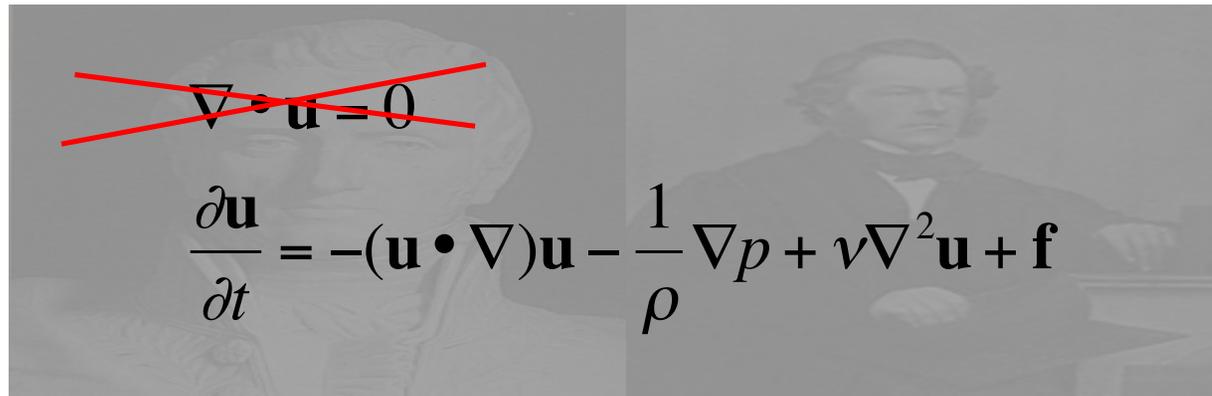
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Have fixed # particles and mass, so...

# SPH

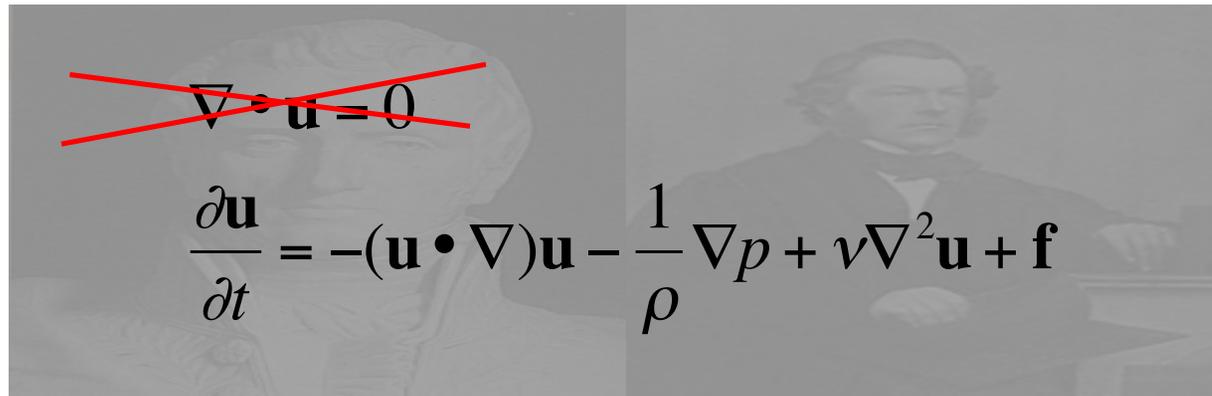
## ● Back to Navier-Stokes

The slide features a background image of a man's portrait, likely a scientist, with a semi-transparent overlay. The equation  $\nabla \cdot \mathbf{u} = 0$  is crossed out with a red double line. Below it, the Navier-Stokes equation is displayed: 
$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

Have fixed # particles and mass, so...  
mass is automatically conserved

# SPH

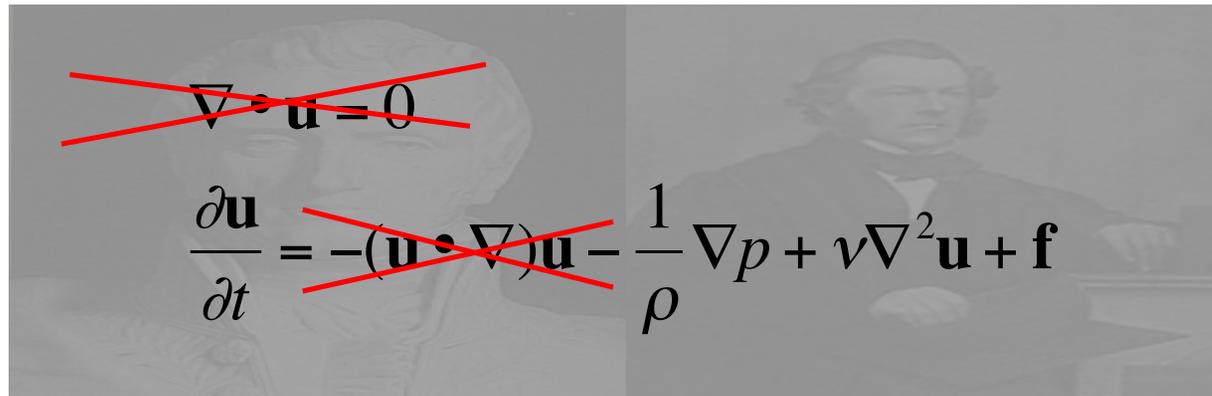
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Advection automagically handled by particle update, so...

# SPH

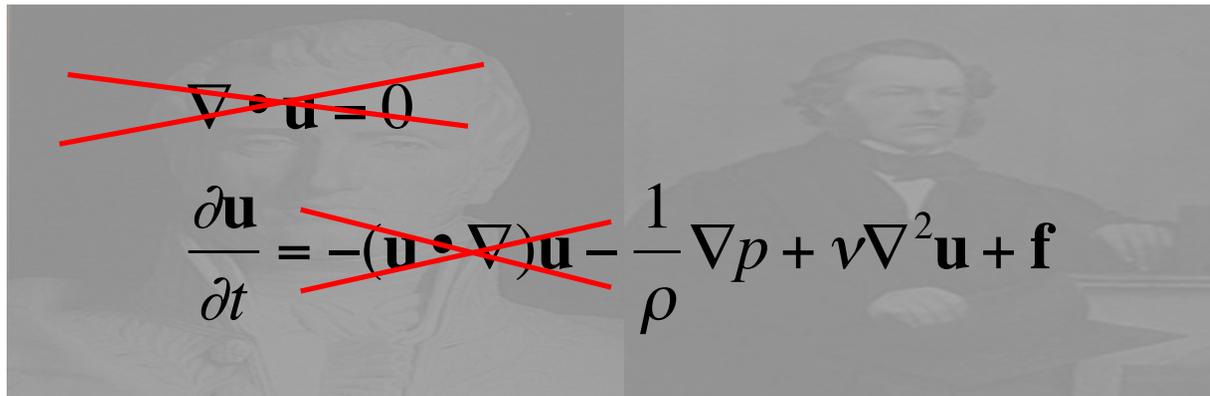
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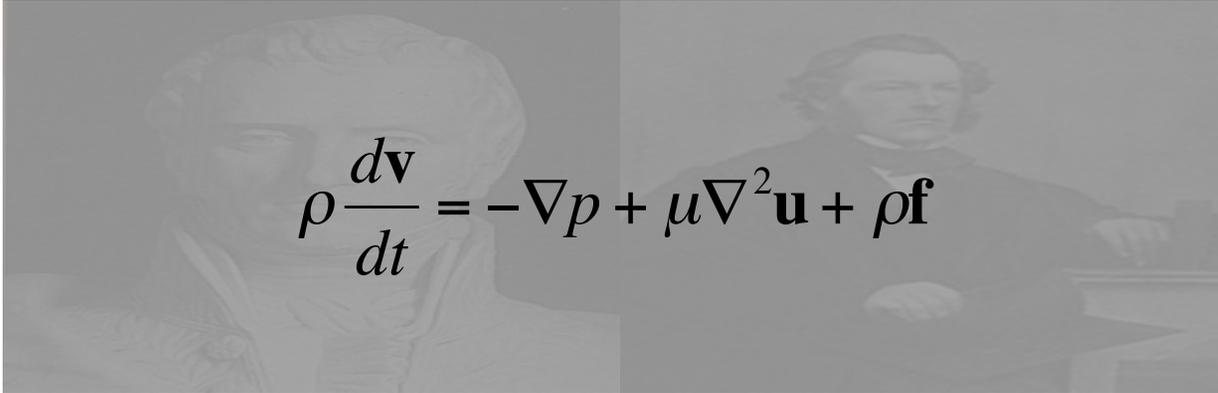
# SPH

- Simplifies to


$$\nabla \cdot \mathbf{u} = 0$$
$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

# SPH

- Simplifies to


$$\rho \frac{d\mathbf{v}}{dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{f}$$

# SPH

- Functional breakdown

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{f}$$

# SPH

- Functional breakdown

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{f}$$

Change in velocity

# SPH

- Functional breakdown

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{f}$$

Pressure

# SPH

- Functional breakdown

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{f}$$

Viscosity

# SPH

- Functional breakdown

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{f}$$

External forces

# SPH

- Compute densities, local pressure
- Generate forces on particles
  - External
  - Pressure
  - Viscosity
- Update velocities, positions
- Render

# SPH

## ● Rendering

- Marching cubes (using smoothing kernel)
- Blobs around particles/splatting

# SPH Implementations

- Takahiro Harada
- Kees van Kooten (Playlogic)
- NVIDIA PhysX
- [Rama Hoetzlein\\*](#) (SPH Fluids 2.0)
- [Takashi AMADA\\*](#)

\* Source code available

# SPH Issues

- Need a *lot* of particles
- Computing level surface can be a pain
- Can be difficult to get stable simulation

# SPH Improvements

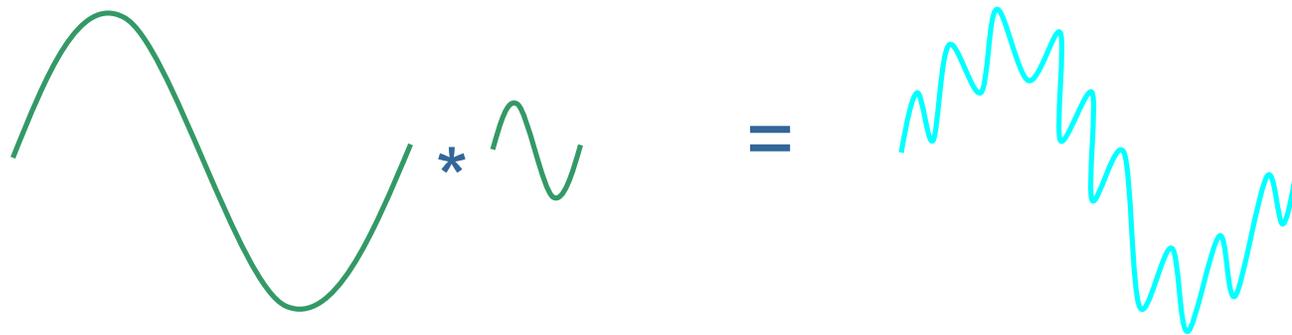
- Spatial hashing
- Variable kernel width
- CFD/SPH Hybrid
  - CFD manages general flow
  - SPH “splashes”

# Surface Simulation

- Idea: for water, all we care about is the air-water boundary (level surface)
- Why simulate the rest?
- This is what Insomniac R20 system does

# R20

- Done by Mike Day, based on *Titanic* water
  - Basic idea: convolve sinusoids procedurally

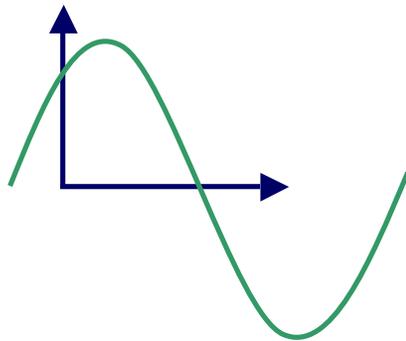


- Much cheaper to multiply in frequency domain and do FFT (assuming periodic)

# R20

## ●Review

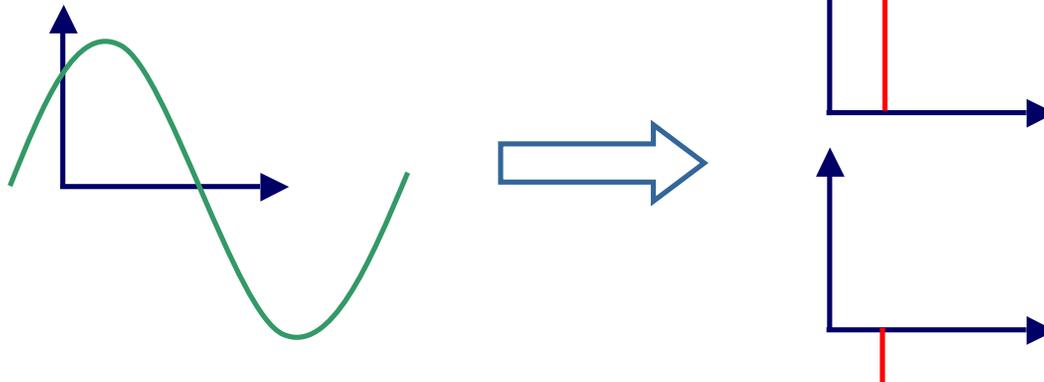
- Sinusoid in spatial domain



# R20

## ●Review

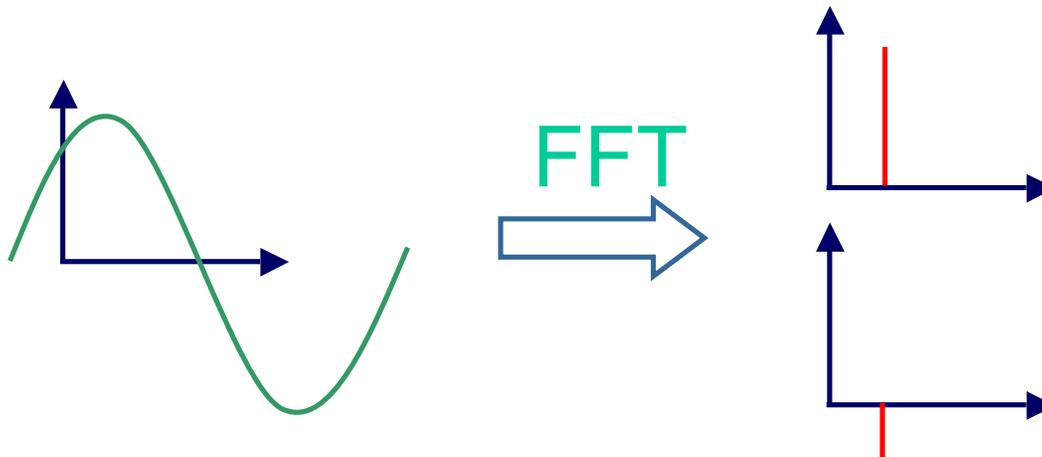
- Can represent as magnitude+phase in frequency slot



# R20

## ●Review

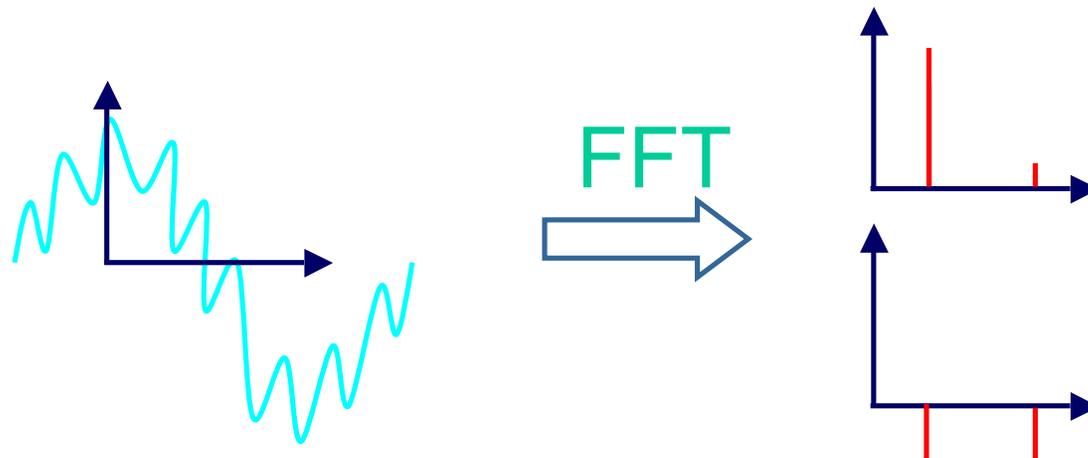
- Requires periodic function



# R20

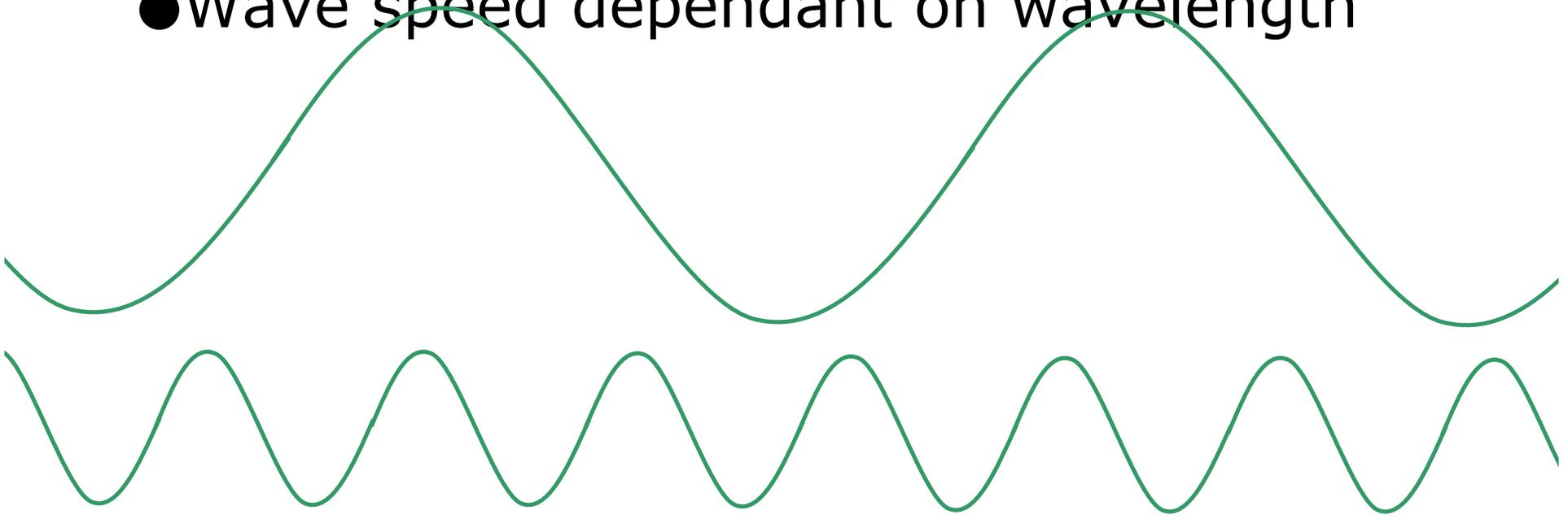
## ●Review

- Multiple sinusoids end up at multiple entries



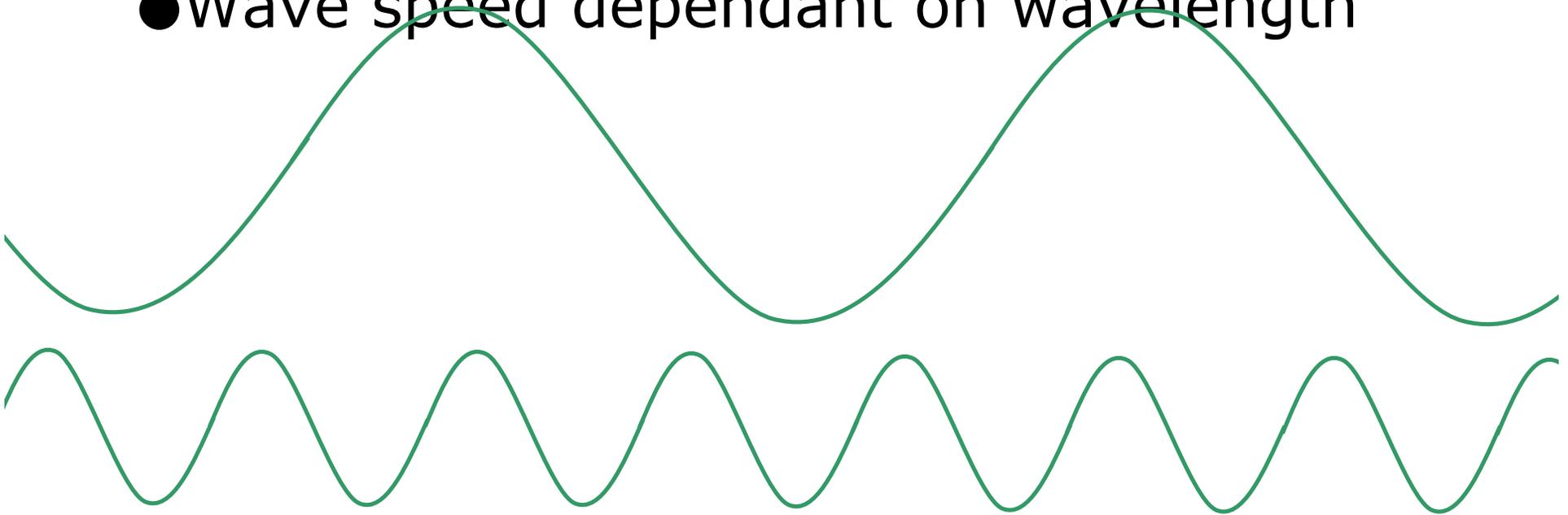
# R20

- Wave speed dependant on wavelength



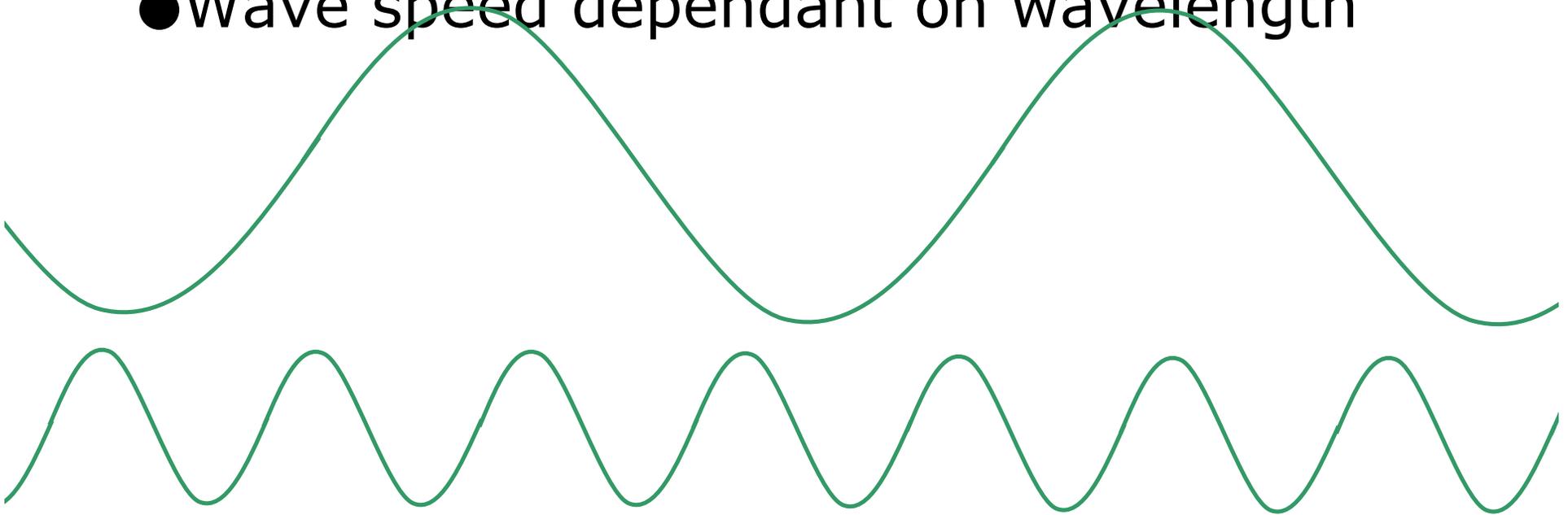
# R20

- Wave speed dependant on wavelength



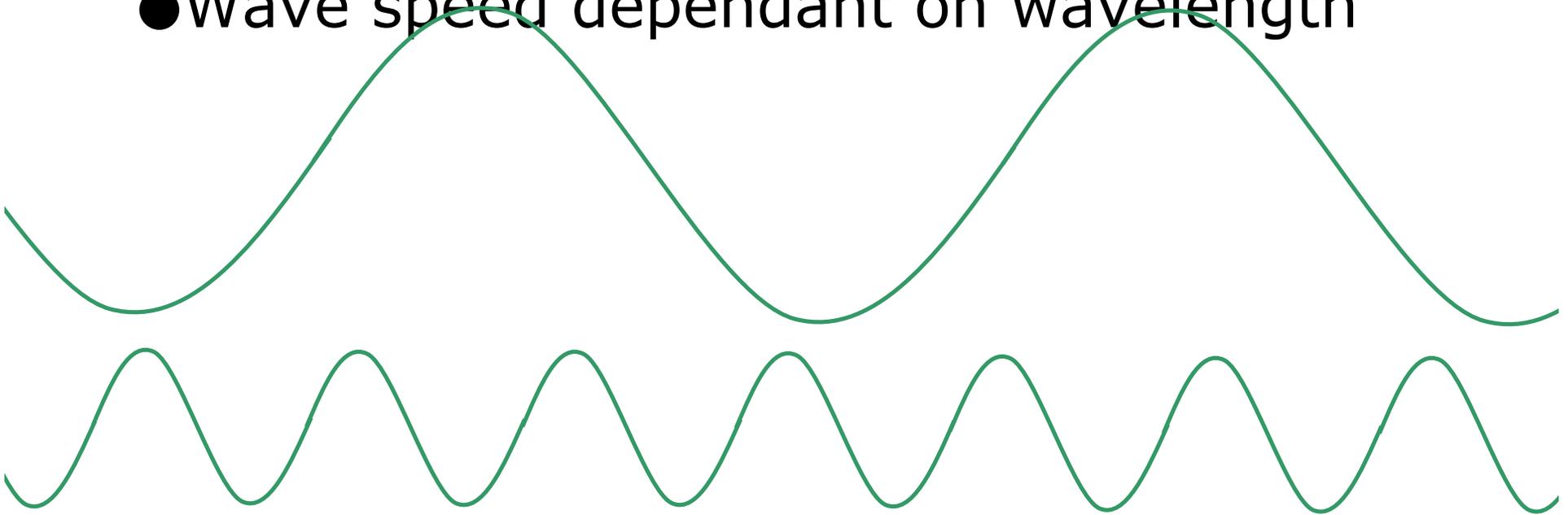
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- Wave speed dependant on wavelength



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- Wave speed dependant on wavelength
  - I.e. phases update at different rates
  - AKA dispersion

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## ● General procedure

- Start with convolved data in  $(r, \varphi)$  form
- Update phase angles for each sinusoid
  - Angular velocity \* dt
  - Dependent on frequency
- Do inverse FFT to get spatial result

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- FFT kernel limited to 32x32
- Combine multiple levels via LOD height field scheme
  - Gives high detail close to camera

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## ● Interactive waves

- Just adding in splashes looks fake
- Instead, do some more FFT trickery so all our work occurs in the same domain
- Non-periodic, so have to manage edges
- Gives nice dispersion effects

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## ●Rendering

- Rendered as height field mesh
- Add normal map for detail
- Cube map/frame buffer map for reflections
- Distortion effect for refractions

# R20

- [Nifty video](#)

# References

- Jos Stam, "Stable Fluids", SIGGRAPH 1999
- Mattias Müller, et. al, "Particle-Based Fluid Simulation for Interactive Applications", SIGGRAPH Symposium on Computer Animation 2003
- Jerry Tessendorf, "Simulating Ocean Water," SIGGRAPH Course Notes.
- <http://www.insomniacgames.com/tech>